# Embedding Cryptol in Higher Order Logic

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# Talk Plan

1. Introduction
2. Existing Embeddings
3. A Natural Embedding
4. Summary

# Cryptol

- Cryptol is a domain specific language for cryptographic applications.
    - Developed by Galois Connections, Inc. since 2002.
- Programs can be executed by the Cryptol (symbolic) interpreter.
- Or compiled to low-level software or hardware.

# Semantics

- What is the meaning of a Cryptol program?
- To use Cryptol as a stepping stone in Evaluation Assurance Level 7 (EAL7) of the Common Criteria, must model Cryptol programs in a formal logic.
- The Cryptol program can then be formally proved equivalent to a specification or low-level implementation modelled in the same logic.

# Higher Order Logic

- Higher order logic is a natural choice for modelling Cryptol programs.
- The type system is a close match with Cryptol's.
- It is a 'wide-spectrum' logic, thus also able to model the specification and/or low-level implementation.
  - No need to defend linking two logics in the evaluation case.
- Example: verifying a Cryptol implementation of elliptic curve cryptography.

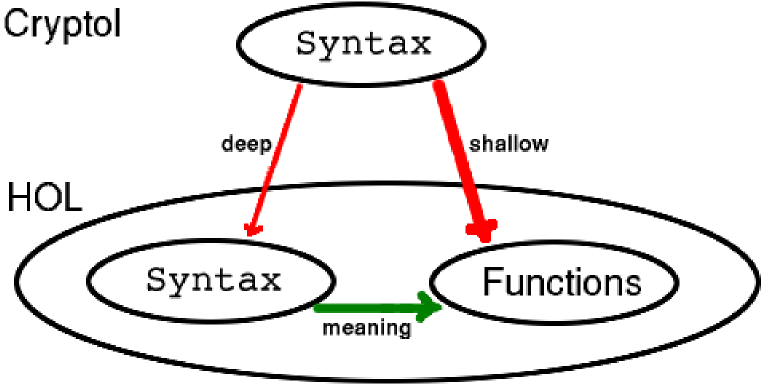
# Semantic Fidelity

- Cryptol has nested mutually recursive sequences.
- These are potentially infinite data structures.
    - OK: HOL4 already has a theory of lazy lists.
- They can contain complex dependencies—evaluating an element might result in program divergence.
    - Warning: Higher order logic functions are total.
- Warning: Cryptol's type system is more fine-grained than higher order logic.

# Verification of Embedded Programs

- Assume we have a semantically faithful embedding of Cryptol into higher order logic.
- How easy is it to prove properties of the embedded programs?
- Rule of thumb: the more 'natural' the embedded programs, the easier to verify.
  - 'Natural' example 1: only terminating Cryptol programs.
  - 'Natural' example 2: encoding sequence length information in higher order logic types.
- Tradeoff: More natural embedding = fewer embeddable Cryptol programs.
  - $\mathrm{SHA1} : [N] \rightarrow [160]$

# Deep and Shallow Embeddings



# Li & Slind (2005)

- Shallow Embedding of Cryptol into HOL4.
- Cryptol sequences are embedded as HOL4 lazy lists.
- Cryptol sequence operations (split, join, etc.) can be uniformly defined in higher order logic.
- Arithmetic operations convert finite subsequences of boolean lazy lists to HOL4 words.
- Syntactic sugar for finite and infinite ranges.
- A definition principle for a particular form of terminating mutually recursive sequences.

# Matthews (2005)

- Deep Embedding of fCryptol into Isabelle/HOL.
- fCryptol is a subset of $\mu$Cryptol.
- Finite and infinite sequences of signed bitvectors have different types.
- Defines an abstract syntax of fCryptol, including mutually recursive sequence definitions.
- The denotational semantics assigns nonterminating sequences a default value.

# Matthews (mid-2006)

- Shallow Embedding of $\mu$Cryptol into Isabelle/HOLCF.
  - HOLCF is an extension of HOL with first-class support for partial functions.
- Mutually recursive sequences are embedded as partial functions.
  - Proving interesting properties require additional proof obligations that expressions terminate.
- Also contains a shallow embedding of the ACL2 logic.
  - Can be used to verify the initial phases of the verifying $\mu$Cryptol compiler `mcc`.

# What is a Natural Embedding?

- What is the most natural embedding of Cryptol into higher order logic?
- Correlated question: How can Cryptol programs be embedded to simplify reasoning about the resulting programs?
- Note that this might involve a severe restriction on embeddable Cryptol programs.
- First step: restrict to terminating Cryptol programs, and embed as native higher order logic functions.
- Second step: how much information can be encoded in higher order logic types?

# Infinite and Finite Length Sequences

- Embed infinite $\alpha$-sequences as

$$\alpha \text{ inf } \equiv \mathbb{N} \to \alpha \ .$$

- Embed finite $\alpha$-sequences of length $n$ as

$$\alpha \text{ vector } \equiv \tau_n \to \alpha$$

  where $\tau_n$ is a specially constructed type having $n$ elements.
- Every sequence in an embedded Cryptol program carries around its length as part of its type.
  - No need for side-conditions about infinite or finite sequence length in theorems: good for verification!

# Sequence Operations

- Using Harrison's finite Cartesian products it's possible to define sequence operations that are polymorphic over $\tau_n$.
- Need finite and infinite versions of the standard sequence operations:

$$\mathsf{seq\_map\_finite} : (\alpha \to \beta) \to [n]\alpha \to [n]\beta$$
$$\mathsf{seq\_map\_infinite} : (\alpha \to \beta) \to [\mathsf{inf}]\alpha \to [\mathsf{inf}]\beta$$

- In practice map to standard Cryptol syntax: the HOL4 parser disambiguates by input argument type.

# Sequence Comprehensions

- Consider a Cryptol implementation of the Fibonacci sequence:
  `fib = [0 1] # [| x + y || x <- drop (1,fib) || y <- fib |]`
- The sequence comprehension can be embedded into higher order logic as

$$\mathrm{map}\ (\lambda(x, y).\ x + y)\ (\mathrm{zip}\ (\mathrm{drop}\ 1\ \mathrm{fib})\ \mathrm{fib})\ .$$

- Print zip using the $\mu$Cryptol symbol $|$, and introduce a new binder syntax for map:

$$(\mathrm{seq}\ (x, y).\ x + y)\ (\mathrm{drop}\ 1\ \mathrm{fib}\ |\ \mathrm{fib})$$

# Mutually Recursive Sequences

- Two step procedure:
  1. Define the sequences as functions $\mathbb{N} \to \alpha$.
  2. Prove them equivalent to the syntax supplied by the user.
- Just an extension of Slind's recursive function definition package TFL.
- Fibonacci example:
  1. $\mathsf{fib}\ i \equiv \mathsf{if}\ i < 2\ \mathsf{then}\ \mathsf{V}[0w;\ 1w]\ \%\%\ i\ \mathsf{else}\ \mathsf{fib}\ (i-1) + \mathsf{fib}\ (i-2)$ .
  2. $\vdash\ \mathsf{fib} = \mathsf{V}[0w;\ 1w]\ \#\ (\mathsf{seq}\ (x, y).\ x + y)\ (\mathsf{drop}\ 1\ \mathsf{fib} \mid \mathsf{fib})$ .
- Compare with the Cryptol implementation:

```
fib = [0 1] # [| x + y || x <- drop (1,fib) || y <- fib |]
```

# Summary

- Motivated and surveyed existing approaches to embedding Cryptol in higher order logic.
- Presented a new approach aimed at simplifying verification of embedded programs.
    - So far only know that it can scale to naturally embed TEA.
- The 'right embedding' will surely depend on the particular reasoning task to be performed, and will borrow ideas from all approaches.