

Here is the result of adding the fact $f\ X\ X$ and the term $f\ \top\ (f\ \top\ \top)$ to a minimal class set:

$$\begin{array}{l} \{ \top \\ \perp \\ f \\ X \\ f\ X \\ f\ \top \\ f\ \top\ \top \\ f\ \top\ (f\ \top\ \top) \end{array} \quad \begin{array}{l} \{ \top, \quad f\ X\ X \}, \\ \{ \perp \}, \\ \{ f \}, \\ \{ X \}, \\ \{ f\ X \}, \\ \{ f\ \top \}, \\ \{ f\ \top\ \top \}, \\ \{ f\ \top\ (f\ \top\ \top) \} \end{array} \quad \{ \}$$

We calculate the matches of our congruence class set:

$$\{ \begin{array}{l} \top \\ \perp \\ f \\ X \\ f X \\ f \top \\ f \top \top \\ f \top (f \top \top) \end{array} \quad \{ \begin{array}{l} \top, \\ \perp \\ f \\ X \\ f X \\ f \top \\ f \top \top \\ f \top (f \top \top) \end{array} \quad f X X \} ,$$

These are the initial matches:

$match(\top, \top)$	$=$	$\{\{\}\}$
$match(\perp, \perp)$	$=$	$\{\{\}\}$
$match(f, f)$	$=$	$\{\{\}\}$
$match(X, X)$	$=$	$\{(X, X)\}$
$match(f X, f X)$	$=$	$\{(X, X)\}$
$match(f \top, f \top)$	$=$	$\{\{\}\}$
$match(f \top \top, f \top \top)$	$=$	$\{\{\}\}$
$match(f \top (f \top \top),$ $f \top (f \top \top))$	$=$	$\{\{\}\}$
$match(X, \top)$	$=$	$\{(X, \top)\}$
$match(X, \perp)$	$=$	$\{(X, \perp)\}$
$match(X, f \top \top)$	$=$	$\{(X, f \top \top)\}$
$match(X, f \top (f \top \top))$	$=$	$\{(X, f \top (f \top \top))\}$

After one inductive step we gain the following match:

$match(f X, f \top) = \{(X, \top)\}$

After two inductive steps we gain the following match:

$match(\top, f \top \top) = \{(X, \top)\}$
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We perform one iteration of the Percolation Algorithm on our congruence class set:

$$\begin{array}{l}
 \{ \top \\
 \perp \\
 f \\
 X \\
 f X \\
 f \top \\
 f \top \top \\
 f \top (f \top \top) \} \\
 \{ \top, f X X \} \\
 \{ \perp \} \\
 \{ f \} \\
 \{ X \} \\
 \{ f X \} \\
 \{ f \top \} \\
 \{ f \top \top \} \\
 \{ f \top (f \top \top) \} \}
 \end{array}$$

The Percolation Algorithm first calculates the match sets, and these would allow it to make exactly one addition: adding $\top$ to the class with representative $f \top \top$. After a congruence closure operation, this is how the classes look:

$$\begin{array}{l}
 \{ \top \\
 \perp \\
 f \\
 X \\
 f X \\
 f \top \} \\
 \{ \top, f X X, f \top \top \} \\
 \{ \perp \} \\
 \{ f \} \\
 \{ X \} \\
 \{ f X \} \\
 \{ f \top \} \}
 \end{array}$$

Examples Chosen to Test the Percolation Algorithm:

Example	Level	Theorem
1	1	$(\forall x. f(f(x)) = g(x)) \Rightarrow (f(g(a)) = g(f(a)))$
2	2	$(\forall x y z. ((x * y) * z = x * (y * z)) \wedge (e * x = x) \wedge (i(x) * x = e)) \Rightarrow (x * i(x) = e)$
3	1	$abc = cba$
4	2	$abcd = dcba$
5	2	$abcde = edcba$
6	3	$(a + 1)(a + 1) = aa + a + a + 1$

Detailed Profiles of the Percolation Algorithm on the Examples:

Ex	It	#C	#E	#M	Match	Assim.	Close	Total
1	1	17	21	26	0.003	0.005	0.007	0.015
	2	15	22	24	0.004	0.005	-	0.009
					0.007	0.010	0.007	0.024
2	1	28	36	46	0.008	0.011	0.012	0.031
	2	28	40	126	0.040	0.125	0.069	0.234
	3	67	101	754	0.738	1.034	0.121	1.893
	4	85	134	857	1.308	1.283	0.079	2.670
	5	76	122	994	1.962	1.138	0.088	3.188
	6	77	129	951	1.738	1.242	0.068	3.048
	7	80	145	857	1.578	1.179	-	2.757
					7.372	6.012	0.437	13.821
3	1	51	70	109	0.027	0.054	0.075	0.156
	2	59	98	118	0.035	0.054	-	0.089
					0.062	0.108	0.075	0.245
4	1	56	75	143	0.033	0.083	0.108	0.224
	2	76	129	1047	0.499	1.348	2.633	4.480
	3	189	401	1168	0.826	1.169	-	1.995
					1.358	2.600	2.741	6.699
5	1	61	80	177	0.045	0.126	0.161	0.332
	2	89	154	1413	0.665	2.053	6.289	9.007
	3	188	477	1562	1.074	1.976	-	3.050
					1.784	4.155	6.450	12.389
6	1	53	72	131	0.033	0.092	0.076	0.201
	2	69	114	770	0.399	1.034	0.957	2.390
	3	83	179	3507	5.929	8.950	10.914	25.793
	4	96	228	2609	5.617	4.995	0.607	11.219
	5	200	406	2501	5.403	6.837	-	12.240
					17.381	21.908	12.554	51.843